Homework on the Greeks

1. **What is the delta of a short position in 1000 European call option on silver futures? The options mature in eight months, and the futures contract underlying the option matures in nine months. The current nine month futures price is $8 per ounce, the exercise price of the options is $8, the risk free interest rate is 12% per annum and the volatility of silver is 18% per annum.**

Delta of a European futures call option is equal to : Exp^(-rT) N(d1)

In this case F0=8, K=8, r=0.12, T=0.6665, sigma =0.18

d1= (ln(8/8)+(0.18^2/2)\*0.6667)/(0.18\*sqrt(0.667))=0.0735

N(d1) = 0.5293

Delta= exp(-0.12\*0.667)\*0.5293=0.4886

Delta of a short position in 1000 futures options is therefore -488.6

1. **In the previous problem, what initial position in nine month silver futures is necessary for delta hedging? If silver itself is used, what is the initial position? Assume no storage cost for silver.**

In order to answer this problem it is important to distinguish between the rate fo the change of the option with respect to the futures price and the rate fo change of its price with respect to the spot price.

The former will be referred to as the futures delta; the latter will be referred to as the spot delta. The futures delta of a nine month futures contract to buy one ounce of silver is by definition1. Hence from the previous problem, a long position in nine month futures on 488.6 ounces is necessary to hedge the option position.

The spot delta of a nine month futures contract is exp(0.12\*0.75)=1.094 assuming no storage costs. (This is because silver can be treated in the same way as a non dividend paying stock when there are no storage costs. F0=S0\*exp(rT) so that the spot delta is the futures delta time exp(rT) ) Hence, the spot delta of the option position is -488.6\*1.094=-534.6. Thus a long position in 534.6 ounces of silver is necessary to hedge the option position.

1. **A financial institution has just sold 1000 seven month European call options on the Japanese yen. Suppose that the spot exchange rate is 0.80 cent per yen, the exercise price is 0.81 cent per yen, the risk free interest rate in the United States is 8% per annum, the risk free interest rate in Japan is 5% per annum and the volatility of the yen is 15% per annum. Calculate the delta, gamma vega, theta and rho of the financial institution’s position. Interpret each number.**

S0=0.80. K=0.81, r=0.08, rf=0.05, sigma=0.15, T=0.5833

d1=(ln(0.80/0.81)+(0.08-0.05+0.15^2/2)\*0.5833)/(0.15\*sqrt(0.5833))=0.1016

d2= d1 – 0.15\*sqrt(0.5833)=-0.0130

N(d1)=0.5405

N(d2)=0.4998

Delta of one call option is e^(-rf\*T)\*N(d1)=exp(-0.05\*0.5833\*0.5405)=0.5250

N’(d1)=exp(-d1^2/2)/(2\*sqrt(pi))=0.3969

Gamma=(0.3969\*exp(-0.05\*0.5833))/(0.8\*0.15\*sqrt(0.5833))=4.206

Vega =S0\*sqrt(T)\*N’(d1)\*exp(-rf\*T)=0.2355

Theta= -S0\*N’(d1)\*sigma\*exp(-rf\*T)/2\*sqrt(T) +rf\*S0\*N(d1) exp(-rf\*T) – r\*K\*exp(-rT)\*N(d2)=-0.0399

rho=K\*T\*e^(-r\*T) N(d2)=0.81\*0.5833\*0.9544\*0.4948=0.2231

1. **A fund manager has a well diversified portfolio that mirrors the performance of the S&P 500 and is worth $360 million. The value of the S&P is 1200 and the portfolio manager would like to buy insurance against a reduction of more than 5% in the value of the portfolio over the next six months. The risk free interest rate is 6% per annum. The dividend yield on both the portfolio and the S&P 500 is 3% and the volatility of the index is 30% per annum.**
2. **If the fund manager buys traded European put options, how much would the insurance cost?**
3. **Explain carefully alternative strategies open to the fund manager involving traded European call options, and show that they lead to the same result.**
4. **If the fund manager decides to provide insurance by keeping part of the portfolio in risk free securities, what should the initial position be?**
5. **If the fund manager decides to provide insurance by using nine month index futures what should the initial position be?**
6. The fund is worth $300,000 times the value of the index. When the value of the portfolio falls by 5% (to $342 million), the value fo the S&P 500 also falls by 5% to 1140.

The fund manager requires European put options on 300,000 times the S&P 500 with exercise price 1140. S0=1200,K=1140,r=0.06,sigma=0.3, T=0.5 and q=0.03.

d1=(ln(1200/1140)+(0.06-0.03+0.3^2/2)\*0.5)/(0.3\*sqrt(0.5))=0.4186

d2=d1-0.3\*sqrt(0.5)=0.2064

N(d1)=0.6622

N(d2)=0.5818

N(-d1)=0.3378

N(-d2)=0.4182

The value of one put option is

1140\*exp(-rT) N(-d2) – 1200 exp(-qT) N(-d1)= 63.40

The total cost fo the insurance is therefore 300,000\*63.0=19,020,000

1. From put call parity

S0\*exp(-qT)+p=c+K\*exp(-rT)

p=c-S0\*exp(-qT)+Kexp(-rT)

This shows that a put option can be created by selling e^(-qT) of the index, buying a call option and investing the remainder at the fisk free rate of interest.

Applying this to the situation under consideration the fund manager should :

1. Sell 360\*exp(-0.03\*0.5) =354.64 million of stock.
2. Buy call options on 300,000 times the S&P 500 with exercise price 1140 and maturity in six months.
3. Invest the remaining cash at the risk free interest rate of 6% per annum.

This strategy gives the same result as buying put options directly.

1. The delta of one put option is exp(-qT)\*(N(d1)-1)=exp(-0.03\*0.5)\*(0.6622-1)=-0.3327

This indicates that 33.27% of the portfolio (i.e.119.77 million) should be insitally sold and invested in risk free securities

1. The delta of a nine month index futures contract is exp((r-q)\*T)=exp(0.03\*0.75)=1.023

The spot short position required is 119,770,000/1200=99,808 times the index

Hence a short position in 99,808/(1.023\*250) =390 futures contracts is required.